Printed Pages : 4 **TAS-104**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9916 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. I) EXAMINATION, 2007-08

MATHEMATICS - I

Time : 3 Hours] *[Total Marks : 100*

Note : *Attempt **all** the problems. Internal choices are mentioned in every problem.*

1 Attempt any **two** parts of the following : **10×2=20**

(a) Define the eigen values, eigen vectors and the characteristic equation of a square matrix. Find the characteristic equation / polynomial, eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} 2 & 5 & 7 \\ 5 & 3 & 1 \\ 7 & 0 & 2 \end{bmatrix}$$

(b) Check the consistency of the following system of linear nonhomogeneous equations and find the solution, if exists :

$$7x_1 + 2x_2 + 3x_3 = 16$$

$$2x_1 + 11x_2 + 5x_3 = 25$$

$$x_1 + 3x_2 + 4x_3 = 13$$

(c) Find the inverse of the matrix

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{5} \\ 1/3 & 1/5 & 1/7 \\ 1/5 & 1/7 & 1/11 \end{bmatrix}$$

2 Attempt any **two** parts of the following : 10×2=20

(a) State Leibnitz theorem for n^{th} differential coefficient of the product of two functions.

If $y^{1/m} + y^{-1/m} = 2x$, prove that

$$\left(x^2-1\right)y_{n+2} + (2n+1)xy_{n+1} + \left(n^2-m^2\right)y_n = 0$$

(b) Verify that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$ where

$$u(x, y) = \log_e\left(\frac{x^2+y^2}{xy}\right)$$

(c) If $u = x\sin^{-1}\left(\dfrac{x}{y}\right) + y\sin^{-1}\left(\dfrac{y}{x}\right)$, find the value of $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$.

3 Attempt any **four** parts of the following : 5×4=20

(a) Expand $e^x \cos y$ about the point $\left(1, \dfrac{\pi}{4}\right)$.

(b) Calculate the Jacobian $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$ of the following :

$$u = x + 2y + z$$
$$v = x + 2y + 3z$$
$$w = 2x + 3y + 5z$$

(c) Discuss the maxima and minima of the function :

$$f(x, y) = \cos x \cos y \cos(x+y)$$

(d) Find a point on the ellipse $4x^2 + y^2 = 4$ nearest to the point $(1, 2)$.

(e) Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $xy + yz + zx = p$.

(f) If $f(x, y) = x^2 y^{1/10}$, compute the value of f when $x = 1.99$ and $y = 3.01$.

4 Attempt any **four** of the following : 5×4=20

(a) Evaluate the following by changing into polar co-ordinates : $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\, dy dx$

(b) Find the area enclosed between the parabola $y = 4x - x^2$ and the line $y = x$.

(c) Change the order of integration in $\int_0^a \int_{x^2/a}^{2a-x} f(x, y) dx dy$

(d) Find the volume of the solid surrounded by the surface $\left(\dfrac{x}{a}\right)^{2/3} + \left(\dfrac{y}{b}\right)^{2/3} + \left(\dfrac{z}{c}\right)^{2/3} = 1$.

(e) Define Gamma and Beta functions. Prove that

$$B(l, m)B(l+m, n)\,B(l+m+n, p) = \frac{\Gamma l\,\Gamma m\,\Gamma n\,\Gamma p}{\Gamma(l+m+n+p)}.$$

(f) Show that $\int_0^1 x^5\left(1-x^3\right)^{10} dx = \dfrac{1}{396}$

5 Attempt any **two** of the following : 10×2=20

(a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then show that

(i) $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$, where $\vec{a}$ is a constant vector.

(ii) $grad\ r = \dfrac{\vec{r}}{r}$

(iii) $grad\ \dfrac{1}{r} = -\dfrac{\vec{r}}{r^3}$

(iv) $grad\ r^n = nr^{n-2}\,\vec{r}$

where $r = \left|\vec{r}\right|$

(b) Prove that

$\vec{a} \times (\nabla \times \vec{r}) = \nabla(\vec{a} \cdot \vec{r}) - (\vec{a} \cdot \nabla)\,\vec{r}$ where $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(c) State the Green's theorem. Verify it by evaluating $\int_C \left[\left(x^3 - xy^3\right) dx + \left(y^2 - 2xy\right)dy\right]$ where C is the square having the vertices at the points $(0, 0)$, $(2, 0)$, $(2, 2)$ and $(0, 2)$.